Low Cost Design for an Orbital Ring

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18/7/2017

Executive Summary

This report outlines a design for an orbital ring with a cost estimate of $9 billion that can be built using existing technology and is well within the resource capacity of over 1200 of the world's largest companies. If this design is verified it will launch a space race to construct the first orbital ring, which could be constructed within a 10 to 30 year time frame. The successful deployment of this orbital ring will reduce the cost of space travel by at least three orders of magnitude to just dollars per kilogram instead of the current $1400/kg.

The main elements of the design include hundreds of metal bolts (40kg each) which travel between ten platforms (4500kg each) that redirect the bolts using superconducting electromagnets and a Tesla coil which is powered by solar panels and later by electricity from the tether. A Zylon tether that is 450km long and weighing 9040kg is dropped from the platforms once the ring has been stabilized. The platforms have direct line of sight between each other enabling the bolt to reach velocities of 30km/s. The entire mass of the ring will be launched into orbit by a single Falcon Heavy rocket lifting a payload of 63.8 tonnes into low Earth orbit. Everything is situated in low Earth orbit which will minimize the accumulated space junk that would result from a complete failure.

Table of Contents

Introduction

The radius and velocity of a circular orbit are related by the following equation.

$$v = \sqrt{\frac{GM_{earth}}{R_{earth}}}$$

Thus, the smaller the radius of the orbit the faster the velocity has to be and vis versa. You can orbit slow enough to match the earth's rotation so the satellite is always above a specific point on earth. This is called geostationary orbit and it is achieved at an orbital radius of 42,000km. The space elevator concept drops a cable from a satellite in geostationary orbit for climbers to shuttle up and down. The problem is there doesn't exist a material that we can currently make which can be that long and survive the necessary forces.

There is another way to get a satellite to hover over a single point on earth and it involves the game of catch. Imagine you have several low earth satellites all equally spaced out in the same orbit. They are orbiting just outside the bulk of the atmosphere about 450km up and are travelling very fast at about 7.5km/s. If one satellite threw a ball to the one behind it at 7km/s (and upwards) that ball would appear stationary to an observer on earth because earth is spinning around at 0.5km/s. The ball is no longer in a stable orbit and will start to fall (lose altitude). But it could be caught by the next satellite which could throw the ball to the satellite behind it. This game of catch can be continued indefinitely and the ball would always be above the same spot on earth. There is a catch however the total orbital momentum has to be the same as a stable orbit so the satellites have to move slightly faster than their normal velocity to make up for the lack of momentum of the ball.

Now imagine that the ball is a platform and the satellites are metal bolts. The metal bolts would pass through the platform redirecting it electromagnetically. You could easily drop a 450km tether from the platform down to earth which climbers could shuttle up and down granting access to space. The larger the total momentum of the metal bolts the more weight could come up the tether.

You need two or more platforms, you'd probably start with at least ten and incrementally add more until you can join them all together to form a complete orbital ring. The same goes for the metal bolts you could start with one (though several hundred would probably be a better starting point) and keep adding more until you have a solid ring of them.

A single Falcon Heavy launch with a total 63.8 tonnes of payload plus the second stage superstructure. The bolts are a solid cylinder of aluminium with a length of 1m and a radius of 7cm, weighing 40kg each. To provide the needed thrust to the platforms the initial orbital ring requires 470 bolts each travelling at 30km/s.

If the ring loses stability the platforms will be destroyed with most of the debris falling down and burning up in Earth's atmosphere. The bolts will still remain in fairly cohesive orbits and depending on their mass and perigees they will also burn up sooner or later. However, it is likely that more platforms could be launched before this happens to capture these bolts and put them in a stable orbit. Thus, even complete failure does not set us back to square one. We will have more resources in space than before we started and will have learned from the cause of failure. The amount of space junk will increase but because everything is in low Earth orbit most of it except for the bolts will burn up in the atmosphere fairly quickly. This ability to fail decreases the risk of the project substantially.

There is a tipping point for the velocity of the bolt. When there are few platforms, the bolts have to curve around earth to reach the next platform. Gravity is used to achieve that curved path but this limits the maximum speed of the bolt. This situation changes when the bolts can take a direct path along the line of sight between the platforms. Because their paths aren't dependent on a specific curvature the bolts can travel at any sufficiently high speed. This means that momentum in this orbital ring can be increased by increasing the velocity of the bolts whereas in the latter situation momentum could only be increased by adding more mass.

Deployment

The following describes a multi-platform deployment using a single rocket for the orbital ring it is just one of many options, perhaps there exists a better alternative.

First the platforms would likely be a cylindrical superconducting electromagnet which the metal bolts pass through. Because the second stage of the rocket will also be cylindrical it could be adapted to function as the platform. This reuse of material increases the bolt percentage of the payload but it will likely require modifications to be made in space. As much of the rocket retrofit will be done on Earth but some will likely have to be done in space if the second stage is reused. Perhaps the best option is to have all the platforms initial dock at the same location in a low Earth orbit then send up astronauts and robots to complete modifications and test systems. Once complete one of the platforms will relocate to the opposite position in the orbit to the rest of the platforms as seen in Figure 1. Note that although it is likely the first orbital ring doesn't have to go around the equator.

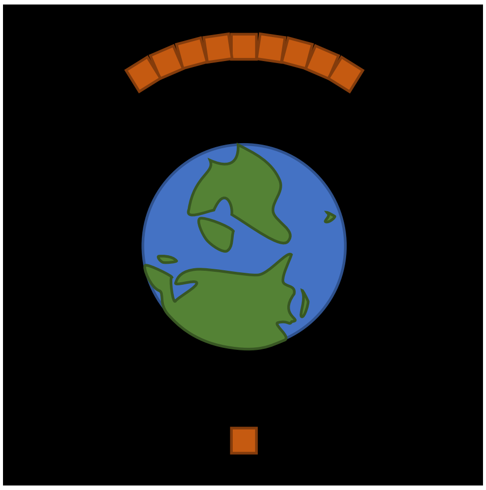

Figure 1: Stage One of Orbital Ring Deployment

Once platform positioning is complete, the electromagnet will launch all of the metal bolts at low speed into an eccentric orbit. Note that this acceleration does not need to be large but rather the bolts can undergo multiple small accelerations to achieve the desired velocity. Platforms are needed at two locations because the first platform will recoil and shift orbit. Because an object in orbit that is given a short acceleration will always return to that point in orbit, the metal bolts will return to the location that they were accelerated. However, the platform due to its recoil will no longer be there. Instead the platform that accelerates the bolts will target the second platform which will then readjust the metal bolt's trajectory to intercept the primary platform's recoiled position as seen in Figure 2.

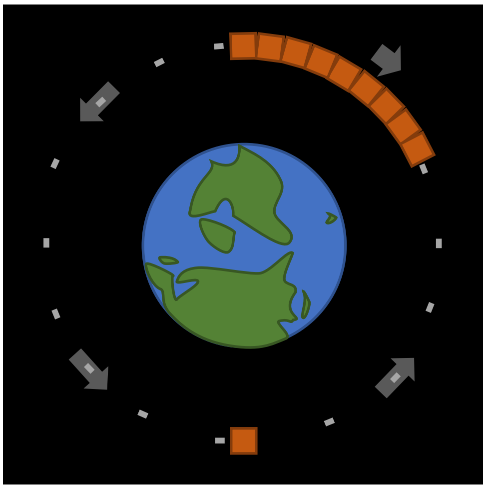

Figure 2: Stage Two of Orbital Ring Deployment

The first platform will launch all of the bolts and will initially be nine platforms linked up end to end. This will enable greater force and precision necessary for the initial acceleration of the bolts. After a stable ring of bolts has been achieved these platforms will disengage and move to equidistant locations on the orbital ring.

Once a ring of bolts with equidistant platforms has been established a tether will drop from each platform and connect to a ground or sea station as seen in Figure 3. Initially this tether will have a limited payload capacity however the payload capacity will increase as metal bolts are added to the ring structure and more tethers are dropped from the platform. To add a bolt, a gap will be made in the ring via coordination between the platforms. The bolt will then be accelerated in an adjacent orbit until it's velocity matches the ring's velocity then it is added to the ring.

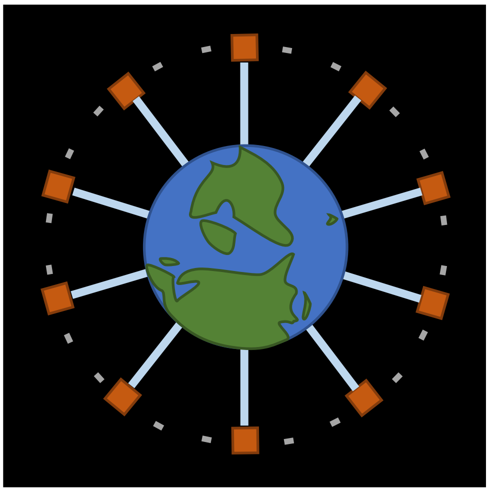

Figure 3: Stage Three of Orbital Ring Deployment

The weight and acceleration of the payload as it travels up the tether will create deviant dynamic forces that threaten the stability of the ring. Real time active readjustment of all the platform's targeting is required to achieve stability. This instability will be the largest for the initial payload but the structure will become more stable as the total momentum of the bolts increases. If a tether is dropped from only one platform that will create a substantial asymmetry in the ring because each tether weighs nine tons. Dropping a tether from all platforms will increase stability but also increase initial required capacity of the system and is the suggested approach.

Platform

The platform has to redirect metal bolts travelling between 0km/s to 30km/s of relative velocity. This has to be done with incredible accuracy and with constant target readjustments. It also has to be able to drop a tether and transfer the tether's payload of bolts into the ring. Furthermore, it has to do all this while being extremely lightweight and powered only by solar energy initially and later by electricity from the tether.

There is a law of difficulty that states difficulty must always be conserved. This orbital ring design basically shifts the engineering difficulty from launching an enormous number of rockets to developing a platform that is a design nightmare. The more powerfully and precise the platform can redirect bolts the fewer rockets will need to be launched.

The platform could have the second stage Falcon Heavy rocket as its superstructure. If this design is chosen the superconducting electromagnet coils would be located inside the second stage. The metal bolts will pass through this cylinder and in the process will be redirected by the magnetic forces. This redirection will provide the upwards momentum needed to keep the platform in orbit. The contact time between the platform and the bolt is 3.3 milliseconds.

The platform design shown in Figure 4 is just one of many potentially viable options. As the bolt approaches the platform it will pass within close proximity to a high voltage electrode that has been charged using a Tesla coil. Electrons will arc from the electrode to the bolt charging it to the required surface charge density. The bolt will then pass through the platform being redirected by the coarse adjustment superconducting electromagnet, the field of this magnet will stay constant. The bolt will then pass through another smaller magnetic field that will fine tune the bolt's direction to precisely target its trajectory. Finally, the bolt will leave the platform passing by an electrode that will remove charge from the bolt, the energy from this discharge will be used to charge the next bolt. This prevents the bolt losing charge as it contacts any molecules in the upper atmosphere (the bolt has a 100km perigee).

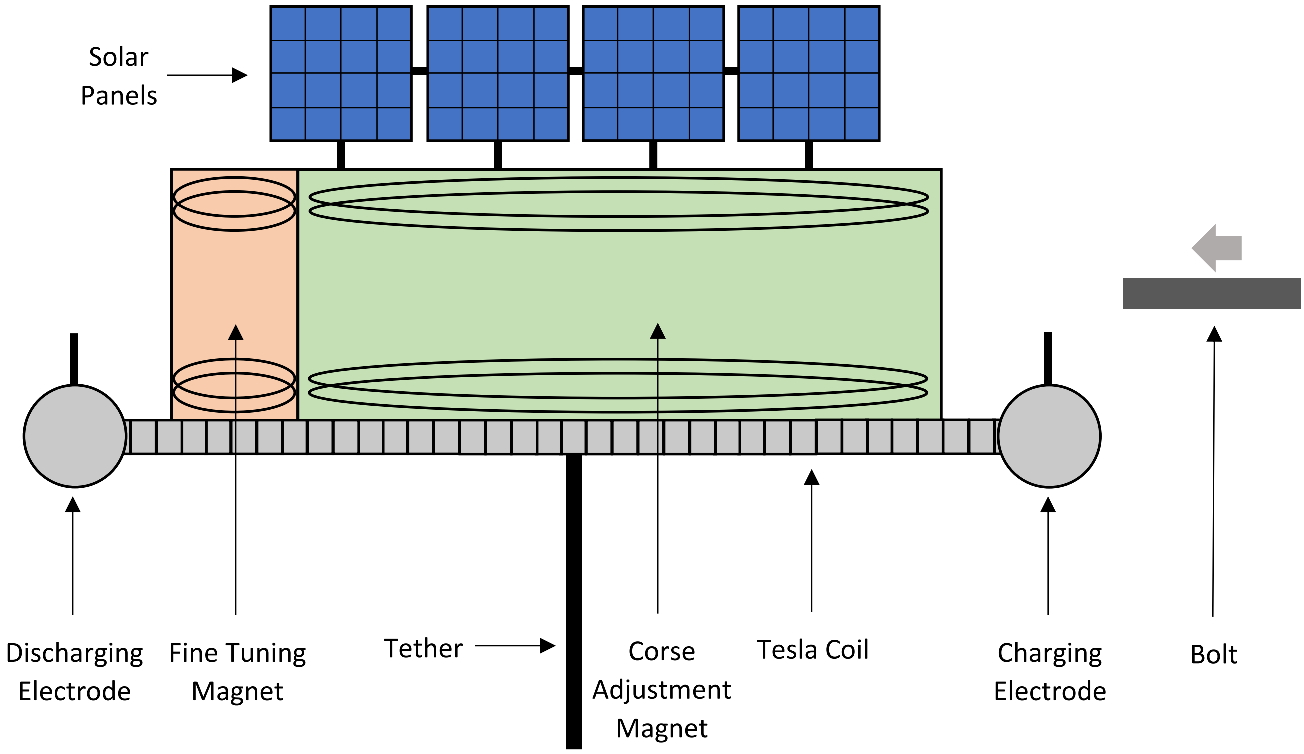

Figure 4: Platform Design Features

Tesla created a coil topped by a sphere with 1.8m^2 of surface area which achieved a charge of between 200 to 600 Coulombs (Childress, 2000). I will make the assumption that the metal bolts can be charged to 300C over a surface area of 0.5m^2. The force on the bolts can be calculated using the Lorentz force law, I will assume the metal bolts travel at right angles to the magnetic field.

$$F = qE + qv \times B$$
$$B = \frac{F_{max}}{q_{bolt} v_{bolt}}$$
$$B = \frac{2.2 \times 10^8 kgms^{-2}}{300C \times 30000ms^{-1}} = 25T$$

The current record for the maximum field strength of superconducting electromagnets at cryogenic temperatures is 32T (Weijers, 2017) so achieving 25T is hard but not out of the realm of possibility. This was calculated for one rocket launch, as is shown in Figure 5 the maximum field strength required greatly reduces with number of rockets. There exists an economic equilibrium between the cost of extra rocket launches and the cost of developing higher specification platforms.

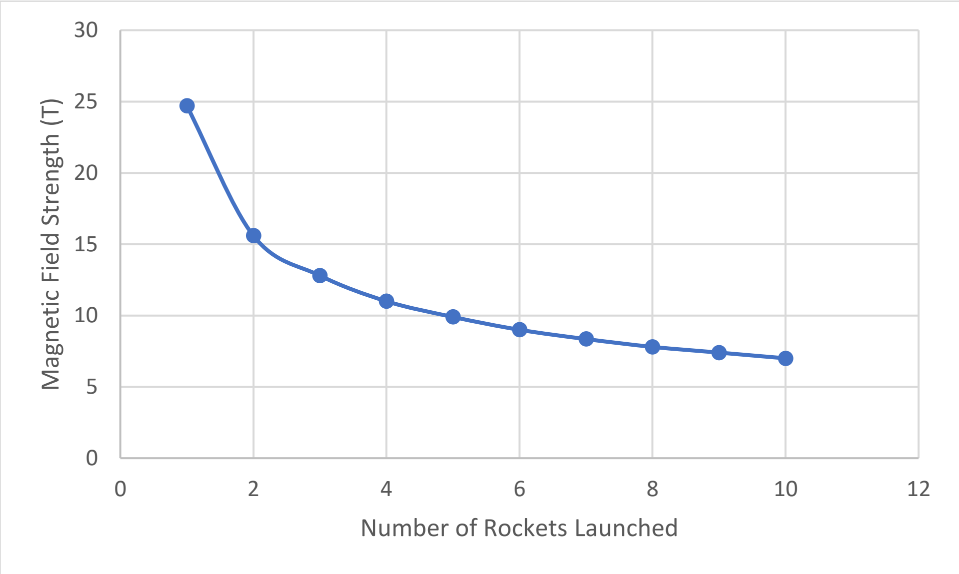

Figure 5: Advantage of Multiple Rocket Launches

The bolt is an aluminium cylinder one meter long, seven centimetres in radius, half a square meter in surface area and weighs 40kg. Aluminium is used because of its low density (which minimizes payload weight), high strength (due to the large forces it has to survive during the redirect), low cost and high conductivity (important to minimize resistive losses during charging and discharging).

There are two steps involved for the platform to add bolts to the ring. First, the platform has to make room in the current ring arrangement by slowing and speeding up some of the bolts in the ring by using adjustments in the fine-tuning section. Second, the platform has to accelerate the bolt to the required velocity in an adjacent orbit this is done using a second accelerator located outside of the second stage of the rocket (not shown in Figure 4). This acceleration can be done over many passes because if an object undergoes a short acceleration in a stable orbit it will always return to the point of that acceleration. Thus, the bolt will always return to the accelerator with limited targeting required. Extra platform stability might be obtained from manipulating the angle that these bolts are accelerated with.

The main superconducting electromagnet will require minimal energy input because there are no resistive losses. The active targeting magnet and the Tesla coil will constitute the largest energy losses of the platform and the engineering challenge is to be able to reduce their power consumption to feasible levels. The power consumption of these devices is estimated below.

The bolts are deflected using two magnets, the static coarse adjustment magnet and the fine tuning magnet that is assumed to be only 0.1% of the strength of the coarse adjustment magnet. The total energy in a magnetic field can be derived from its field strength and volume as follows.

$$B_{energy} = \frac{1}{2}\frac{Volume \times B^2}{\mu}$$

If the fine tuning magnet's magnitude varies from zero to full power between the passage of each bolt then the magnet consumes 26MW of power. This power load significantly drops if more rockets are used to 6MW if ten Falcon Heavies were launched.

The Tesla coil charges the bolt, which then travels through the magnets and gets discharged before it leaves the platform. The discharging step might not be necessary if the upper atmosphere doesn't strip charges of the bolt's surface as it passes through it at its perigee (100km altitude). However, assuming that discharging is necessary I'm not sure of the best way to estimate the power requirements of this process. I will assume that the cylindrical bolt inside the cylindrical magnet can be modelled as a coaxial capacitor with the following capacitance.

$$C = \frac{q}{V} = \frac{2\pi\epsilon l}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$V = \frac{q ln\left(\frac{R_2}{R_1}\right)}{2\pi\epsilon l}$$

Where R_1 is the radius of the bolt and R_2 is the radius of the hole in the platform that the bolt passes through. The work stored in a capacitor is.

$$W = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

This work has to be done every time a bolt passes through enabling the estimation that a required 2×10^{14}W of power is needed. This is enormous and almost certainly incorrect. It is roughly the same order of magnitude as the global annual energy usage. It is useful to note that almost all of the energy to charge a bolt can be obtained from the discharge of the previous bolt which could result in a feasible operational power consumption.

Although the total energy required to support a tether is enormous the system can build up this total energy over long periods of time from a very small amount of required initial energy. Also, the operational power consumption can theoretically be zero if no active targeting is required and no losses occur during charging/discharging. Thus, the operational power consumption is dependent on how well the platform is engineered. Lastly this design has not been optimized and these models might be significantly off, there is a lot of room for improvement.

Before the tether is dropped this energy will have to be delivered from space based energy generation. Solar panels are the best option using current technology. They can achieve a power to weight ratio of 2kW/kg (Shiu, Zimmerman, Wan, & Forrest, 2009) and when located in space solar panels have a higher collection rate, a longer collection period and need less support structure. If this ring is situated around the equator that means some platforms will be in Earth's shadow shading their solar panels. However, these platforms could obtain energy from slightly slowing down the bolts moving through them. If that doesn't work then there are ring arrangements that ensure all platforms are in the sun but these tend to be suboptimal from a practical point of view. Energy could also be obtained from a nuclear fission reactor but this complicates the design and increases the potential risk considerably particularly from radioactive fuel being spread across the upper atmosphere in the event of a rocket failure. Fusion of course would be ideal however it is future technology and it seems that fusion powerplants will not have the power to weight ratio needed. This could be overcome by placing the fusion powerplants in a stable orbit adjacent to the orbital ring and transmit power wirelessly to the platforms as they pass by. This approach can also be used with solar powerplants too. However, after the tether is dropped electricity from a ground station can be sent through the tether (this would also power the climber). Zylon the proposed tether material can be made electrically conductive.

Tether

The tether will connect the platform to the ground station. The tether will be dropped when a stable ring of bolts has been achieved. The stresses on the tether are not constant because each part of the tether has to hold up the weight of the tether below it. Thus, the tether will taper according to the below equation that was derived by Aravind in his 2006 paper titled "The physics of the space elevator".

$$\frac{A_{top}}{A_{bottom}} = \mathrm{e}^{\frac{\rho g R_{earth}}{2T}\left(\left(\frac{R_{earth}}{R_{earth}+R_{apogee}}\right)^3-3\left(\frac{R_{earth}}{R_{earth}+R_{apogee}}\right)+2\right)}$$

This equation was numerically integrated over the length of the tether to determine the total tether mass. This was done for three materials; Kevlar, Dyneema and Zylon, for a tether length of 450km and a bottom tether radius of 2mm, the results are shown in Table 1. Due to stresses such as wind loading, icing, water condensation and other stresses, the climber mass plus payload will not exceed half the maximum climber mass stated in the table.

Minimizing the tether mass will minimize the initial capacity needed in the orbital ring however maximising the climber mass increases the tether strength and increases the rate at which mass can be added to the orbital ring. These considerations are more important than the tether cost which will be less than 1% of the total orbital ring cost.

Because Zylon has the highest tether mass to maximum climber mass ratio it is the best option. Zylon also has many amenable properties for this application such as it has the highest specific strength of any commonly made material, it has good thermal stability, it is also flame resistant and can be plated with metals to become electrically conductive. This last property means that the tether could provide electrical energy from the ground station to the climber and the platform.

Table 1: Material Options for Tether

MATERIAL	DENSITY ρ (KG/M^3)	MAX TENSILE STRESS T (GPA)	TETHER MASS (KG)	MAX CLIMBER MASS (KG)	TETHER COST ($USD)
DYNEEMA	970	3.60	5700	4610	$77,000
KEVLAR	1440	3.62	8620	4630	$220,000
ZYLON	1540	5.80	9040	7430	$3,800,000

Assuming a tether length of 450km, the tether radius at the bottom to be 2mm and a constant value for g of 9.81$m \cdot s^{-2}$.

Cost

These costs are order of magnitude estimates which aim to obtain a ball park figure. The breakdown of the cost can be seen in Figure 6 and all values are in US dollars. The total cost is $8.9 billion of which $90 million is needed for the Falcon Heavy rocket lifting 63.8 tonnes of platforms and the bolts into low Earth orbit. $6 billion is needed for the ten platforms including their power generation, assuming each platform costs the same as an average satellite ($300 million each) with research and development doubling the costs to $600 million each (Globalcom, 2016). $5 million will be for the bolts and tether with the tether costing $3.8 million and the bolts made from aluminium with a price of $1.9/kg (Investment Mine, 2017) with 19 tonnes used resulting in a total cost of $40,000 plus an extra $1 million for the design and manufacture of the bolts and tether. $2.5 billion is needed for the design and construction of the ground stations and the climbers. A 25% margin will be included to cover the miscellaneous as well as to cover any cost blow outs or failures.

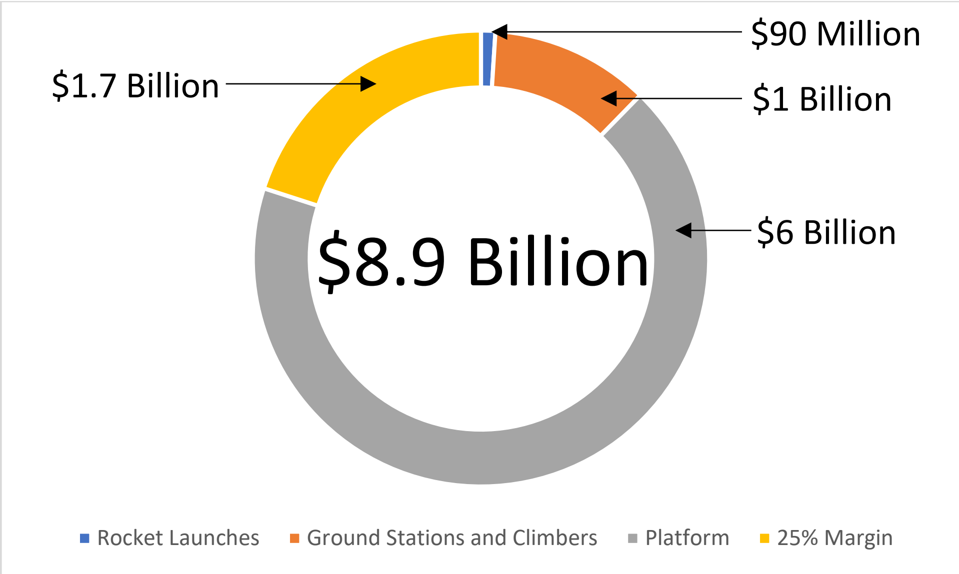

Figure 6: Orbital Ring Costing (numbers may not add to total due to rounding)

The total price is about $9 billion, this is an order of magnitude estimate meaning the cost probably won't be over $100 billion but is unlikely to be less than $1 billion. There are many entities that have to resources to fund a $9 billion project such as the approximately 140 individuals with a net worth higher than this (Elon Musk is one of those) (Forbes, 2017). Nasa could easily afford this with their annual budget of $19.5 billion in 2017. Also, there are over 1200 public companies that have over $9 billion in assets (Forbes, 2017). And the top 20 largest countries by GDP would only have to put aside about 1% or less of their budgets to fund this project (Statistics Times, 2016). If this design is verified then there will be a space race between many of these entities to develop the first functional orbital ring.

An orbital ring space race could be catalysed by a multi-stage X Prize. X Prizes leverage many small innovative teams to compete for prize money that is only paid to the winner effectively multiplying funding impact while increasing publicity. Intermediate quantitative goals could break down the total technical challenge, two examples of goals could include achieving a desired magnetic field strength to weight ratio and achieving an operational Tesla coil and bolt design that can transfer the necessary charge within a strict power limitation.

Conclusion

This report presents a new design of an orbital ring and covers the most difficult and unique aspects of its deployment. This initial analysis hasn't identified any physical impossibilities (deal breakers), or the need for any future technology but it has shown that this will be a considerable engineering challenge. If the design is verified then an order of magnitude cost estimate reveals it would cost about $9 billion well within reach of thousands of entities globally. If the design is verified it will launch a space race that will accelerate progress possibly completing the first orbital ring in a 10 to 30 year time frame.

This first orbital ring will enable the construction of other stronger rings. The direct potential benefit will be to lower the cost of space travel from $1400/kg (SpaceX, 2017) to dollars per kilogram. This will make settlement of the Moon and Mars inevitable and will provide cheap power and fast internet to everyone on the planet. Also, long distance transportation between continents will take minutes instead of hours. The ring would be the single largest factor ever to unify the planet and promote globalisation and would lead to the empowerment of everyone in their everyday lives. The importance of an orbital ring cannot be understated and the fact that it is within our current abilities brings me great hope for the future.

Although the crucial elements of the design have been sketched out several important but less unique aspects still need to be worked out such as the climber and the ground station. Those aspects that have been sketched out need considerable development. Fortunately, much of the work on space elevators can be transferred to this project. There are also many potential deal breakers some of which are listed below that have yet to be investigated.

- Solar panels won't be able to provide enough power to meet the energy needs of the platforms.
- Atmospheric loading on the tether will be too much.
- Even with active precision targeting the ring is unstable.
- It isn't possible to achieve the necessary magnetic field strengths within the limited mass and power constraints of the platforms.

I would like your help to investigate the potential deal breakers, develop this design, to double check my calculations and just in general verify the feasibility of this design. This is the reason I have attached my calculations in the appendix and an Excel spreadsheet modelling the orbital ring. Together we can make the orbital ring a reality or at least find out why it doesn't work.

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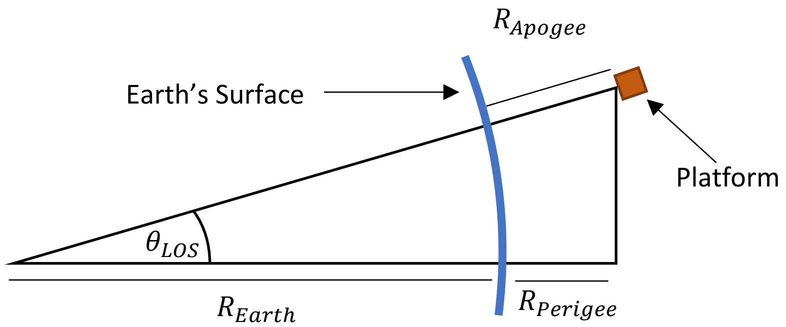
Appendix

Below is a sample of the working for the calculations performed in the Excel spreadsheet used to generate the values found in the body of the report.

Calculation 1: Determining the number of platforms required to obtain line of sight (LOS)

When the bolts have to use Earth's gravity to curve around to get from platform to platform their velocity is fixed by the point of apogee and perigee. However, if there is a direct line of sight from platform to platform then the bolts no longer need to curve because of gravity and they can travel any sufficiently high speed. The number of platforms required to achieve this depends on the apogee and perigee points and is simple to calculate as shown below. θ_{LOS} is the angle for line of sight.



$$\theta_{LOS} = \arccos\left(\frac{R_{Earth} + R_{Perigee}}{R_{Earth} + R_{Apogee}}\right)$$

$$No. of\ Platforms = \frac{360}{2\theta_{LOS}}$$

Calculation 2: Calculating the angle of bolt deflection (α) for a bolt trajectory without line of sight

The equation that describes the elliptical orbit is

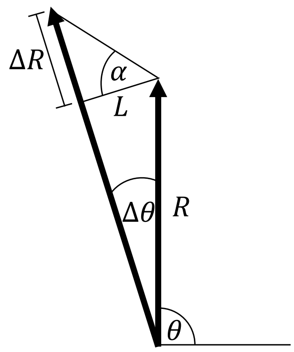
$$R = \frac{a(1 - e^2)}{1 + ecos\theta}$$

R and θ are known for two unique points on the orbit; at the point of perigee and at the platform. This results in two equations which can be used to solve for variables a and e.

$$1.)\ a = \frac{(R_{perigee} + R_{earth})(1 + e)}{1 - e^2} = \frac{(R_{apogee} + R_{earth})(1 + ecos\,\theta)}{1 - e^2}$$

$$e = \frac{\dfrac{R_{apogee} + R_{earth}}{R_{perigee} + R_{earth}} - 1}{1 - \dfrac{cos\theta(R_{apogee} + R_{earth})}{R_{perigee} + R_{earth}}}$$

Variable a can be found by substituting the value of e into equation 1.). Now that the orbit is fully described the angle of deflection (α) can be calculated by the following.



$$\sin(\Delta\theta) = \frac{L}{R}$$
$$L = Rsin(\Delta\theta) \approx R\Delta\theta$$
$$\tan(\alpha) = \frac{\Delta R}{L}$$
$$\tan(\alpha) = \frac{\Delta R}{R\Delta\theta}$$
$$\therefore \frac{\Delta R}{\Delta\theta} = R\tan(\alpha)$$
$$2.)\ \frac{dR}{d\theta} = R\tan(\alpha)$$

To solve this derivative use the chain rule.

$$\frac{dR}{d\theta} = \frac{dR}{du}\frac{du}{d\theta}$$
$$Let\ u = 1 + ecos\theta$$
$$\frac{dR}{du} = \frac{d}{du}\left(\frac{a(1-e^2)}{u}\right)$$
$$\frac{dR}{du} = -\frac{a(1-e^2)}{u^2}$$
$$\frac{du}{d\theta} = -esin\theta$$
$$3.)\ \frac{dR}{d\theta} = \frac{aesin\theta(1-e^2)}{(1+ecos\theta)^2}$$

Sub 2.) into 3.)

$$R\tan(\alpha) = \frac{aesin\theta(1-e^2)}{(1+ecos\theta)^2}$$
$$\alpha = \arctan\left(\frac{aesin\theta(1-e^2)}{R(1+ecos\theta)^2}\right)$$

For two platforms $\theta = 90°$ and the expression for α simplifies to the following.

$$\alpha = \arctan\left(\frac{ae(1-e^2)}{R}\right)$$

The velocity of bolt when it passes through the platform is calculated from the Vis-viva equation below.

$$v_{bolt} = \sqrt{Gm_{earth}\left(\frac{2}{R_{earth}+R_{perigee}} - \frac{1}{a}\right)}$$

Calculation 3: Calculating the momentum transferred to the platform

Calculate the momentum transferred from the metal bolts to the platform.

$$\rho_{bolt} = mv = 40\times 10000 = 4\times 10^5 kgms^{-1}$$

$$Time\ the\ bolt\ takes\ to\ pass\ through\ the\ platform = T_{bolt} = \frac{L_{platform}}{v_{bolt}} = \frac{50}{10000} = 0.005s$$

Because there is line of sight between platforms use the deflection angle calculated in calculation 1.

$$F_{thrust} = \frac{d\rho}{dt} = \frac{\Delta\rho_{bolt}}{T_{bolt}} = \frac{2\rho_{bolt}\sin\theta_{LOS}}{T_{bolt}} = \frac{2\times 4\times 10^5\times \sin(18)}{0.005} = 1.5\times 10^8 N$$

This is the thrust applied to the platform when a bolt passes through however it is useful to calculate the average thrust applied to platform which would use the time interval between the bolts ($T_{interval}$).

$$F_{avg\ thrust} = \frac{\Delta\rho_{bolt}}{T_{interval}} = \frac{2\times 4\times 10^5\times \sin(18)}{0.48} = 5.1\times 10^5 N$$

$$Lifting\ mass\ margin = \frac{F_{avg\ thrust}}{g} - M_{tether} - M_{climber} - M_{platform}$$

$$Lifting\ mass\ margin = \frac{5.1\times 10^5}{9.81} - 9040 - 7430 - 4500 = 8.2\times 10^3 kg = 8.2 tonnes$$

The attached Excel spreadsheet has the complete calculations including the calculations for the mass of the tether and climber.

Calculation 4: Determining platform freefall distance between bolts

Calculate the time interval between metal bolts. Where T_{period} is the time it takes for the bolt to circumnavigate the globe and $T_{interval}$ is the time between one bolt and the next.

$$T_{period} = \frac{2\pi(R_{earth} + R_{apogee})}{v_{bolt}} = \frac{2\times\pi\times(6.37\times 10^6 + 4.5\times 10^5)}{10000} = 4290s = 71min$$

$$T_{interval} = \frac{T_{period}}{N_{bolts}} = \frac{4290}{12200} = 0.35s$$

Calculating the distance of platform free fall with constant g.

$$a = -g = \frac{dy}{dt}$$

$$\int dv = \int -gdt$$

$$v = -gt + v_0$$

$$\frac{dR}{dt} = -gt + v_0$$

$$\int dR = \int (-gt + v_0)dt$$

$$R = -\frac{1}{2}gt^2 + v_0 t + R_0$$

$$\Delta R = -\frac{1}{2}gt^2 + v_0 t$$

Now substitute the following.

$$g = 9.81ms^{-2}$$
$$t = T_{interval} = 2.7s$$
$$v_0 = 0$$
$$R_0 = R_{earth} + R_{apogee} = 6.47\times10^6 m$$

$$\Delta R = -\frac{1}{2}\times9.81\times0.35^2 = -0.6m$$

The platform will oscillate about 30cm between bolts without damping. This is can be corrected for and is not a significant factor.